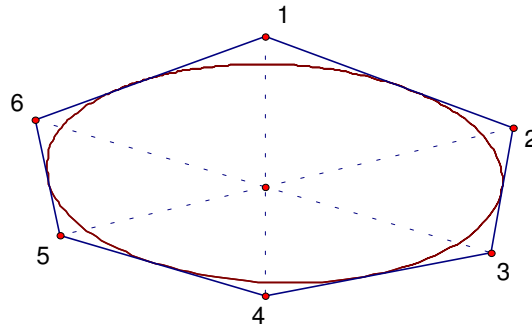


62. Brianchon's Hexagram Theorem.

Prove that the three opposite lines of a hexagram circumscribed about a conic section pass through a point.



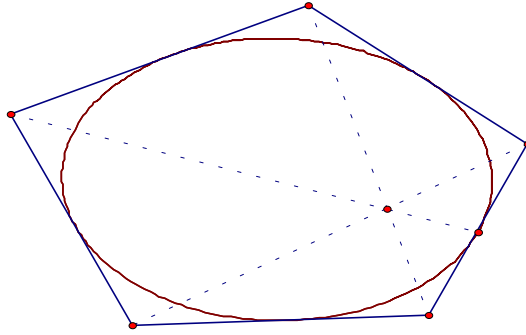
A hexagon circumscribed about a conic section consists of six tangents to the conic section. If adjacent tangent lines intersect in points 1, 2, 3, 4, 5, 6, we get a hexagon 123456 with opposite vertices 1 and 4, 2 and 5, and 3 and 6. Brianchon's Theorem asserts that the lines 14, 25 and 36 are concurrent. This theorem was first published in 1810 by the French mathematician Charles-Julien Brianchon (1785-1864) in the *Journal de l'École Polytechnique*.

Dörrie presents a projective proof very similar to the proof of Pascal's Theorem in No. 61. This should come as no surprise, since the real projective plane obeys the principle of duality which states that any theorem remains true when "point" and "line" are interchanged (and suitable changes in language are made), and Brianchon's Theorem is the dual of Pascal's Theorem.

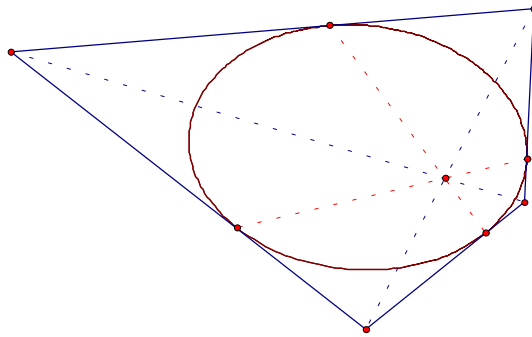
Even a cursory read of Dörrie in German or Antin's translation makes this duality clear. Thus all the proofs (of lemmas, theorems and corollaries) are omitted. However, I will state and illustrate some of them.

The Converse of Brianchon's Theorem. If lines 14, 25 and 36 of a hexagon 123456 are concurrent, the sides are tangent to a conic section.

Corollary 1. (of Brianchon's Theorem) In every pentagon circumscribed about a conic section, the lines joining two pairs of nonadjacent vertices and the line through the fifth vertex and the point of tangency with it opposite side are concurrent.



Corollary 2. In every quadrilateral circumscribed about a conic section, the two diagonals and the two lines joining points of tangency on opposite sides are concurrent.



Corollary 3. In every triangle circumscribed about a conic section, the lines connecting the vertices with points of tangency of the opposite sides are concurrent.

